Investing in Global Hard Assets:
A Diversification Tool for Portfolios

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Introduction

Investors used to take comfort in the notion that a portfolio diversified among domestic stocks and bonds would provide sufficient returns at the price of only moderate risk. There was good reason for this comfort. Investors have been aware of the important role that correlation between portfolio components plays in determining the risk of a portfolio at least since the development of mean-variance optimization by Markowitz [1952]. The lower the correlation, the better, which used to be exactly what domestic stock and bond investors experienced. From 1926 to 1969, the correlation between annual total returns for U.S. stocks and bonds was an attractive -0.02. Today, U.S. stock and bond markets mostly move in the same direction. This tendency is reflected in the correlation that was 0.23 from 1970 to 1980 and 0.58 from 1981 to 1998. This lack of diversification, in combination with attractive returns observed in other asset classes, drives the vigor with which opportunities in non-traditional (or alternative) asset classes have been pursued in recent years.

One of the more heavily researched alternative asset classes is commodities. A difficulty with the previous studies is that the indexes used to characterize the asset class are usually composed of returns from managed futures accounts or passive positions in various commodities.[1] Unfortunately, these investments are available only to institutions or the wealthiest of individual investors.

Most mutual funds in operation that seek to exploit this asset class do not, in fact, invest exclusively through managed futures or passive commodity positions. The goal of this study is to determine what type of role this asset class (given how funds actually invest) has in a strategic asset allocation. In making this determination we have created a Global Hard Assets Index (GHAI). The returns to this index are composed of two components.

1. Returns to the stocks of companies from around the world whose primary business is linked to the production, extraction, or sale of hard assets and

2. Returns to the commodities themselves.

Our results indicate that for the aggressive (i.e., all equity) investor, an allocation to hard assets of up to 25 percent is warranted, with at least a 10 percent allocation suitable for moderate investors.[2] These results are primarily driven by the low correlation coefficients that the GHAI has with other asset classes.

The balance of this paper explains the construction of the GHAI, provides data on the historical performance of the index and other asset classes, develops our inputs for mean-variance optimization, and reports the role that hard assets play in an efficient portfolio.

Defining Hard Assets

While financial assets garner the most attention by investors, non-financial (i.e., real or tangible assets) actually constitute the majority of world wealth. Some types of assets are difficult to place in either the financial or non-financial category, but in general non-financial assets tend to have intrinsic value (i.e., value in use). Typically this use is in some sort of manufacturing process or as a consumable.

Real assets may be divided into "hard" and "soft" assets. Hard assets are non-perishable real assets and include real estate and commodity-related assets such as energy (e.g., oil and gas), precious metals (e.g., gold and silver), industrial metals (e.g., aluminum and copper), and timber. "Soft" assets are perishable and consumable and include the commodities of agricultural products and livestock.

Index Construction

The components of the GHAI were selected in order to represent the types of assets held in mutual funds that seek to exploit profitable investment opportunities in this area. Very few of the many "precious metals" or "natural resource" mutual funds invest in actual commodities, preferring instead to purchase shares of companies involved in the production and servicing of commodities. Mutual funds act in this way, in part, because they are subject to extensive regulation under federal and state securities laws, the Commodity Futures Trading Commission, and federal tax law. Taken together these regulations effectively require a fund to have no more than 25 percent of its portfolio invested in commodities and commodity futures contracts. For example, certain states permit no more than 25 percent of a fund's assets to be committed to commodity contracts. The Internal Revenue Code of 1986 limits a fund's ability to generate short-term gains on assets held for less than three months or from gains on commodities.

To reflect these realities as to how individual investors (through mutual funds) can actually get exposure to this sector, the GHAI is weighted such that 75 percent of the index is invested in equities and 25 percent in commodity futures. The equity and commodity pieces are further broken down into subindexes representing particular types of companies and commodities. These subindexes and their weightings in the overall index are given in Table 1.

The GHAI focuses solely on hard asset commodities because they are more of a long-term store of value and there is a relatively fixed supply. Furthermore, agricultural commodities (soft assets) are greatly affected by weather. Hard assets are independent of these effects and provide a more reasonable hedging vehicle.

There are three interesting aspects about this index.

1. Using the MSCI subindexes means that the equity portion is largely a global index. This implies the potential for capturing some of the diversification benefits that have accrued to the U.S. holders of international equities. The MSCI real estate series is not included because it is dominated by Asian property management companies. The NAREIT series is used as it better represents real estate investments held by U.S. investors.

Table 1: Components and Weightings of Global Hard Assets Index

Components	Weights	Start Date of Index
Morgan Stanley Capital International Gold Mines Subindex	15	1970
Morgan Stanley Capital International Non-Ferrous Metals Subindex	15	1970
Morgan Stanley Capital International Energy Sources Subindex	15	1970
Morgan Stanley Capital International Forest Products and Paper Subindex	15	1970
National Association of Real Estate Investment Trusts Equity Index	15	1972
Subtotal of Equity Component	75	
Goldman Sachs Energy Index	8 1/3	1983
Goldman Sachs Precious Metals Index	8 1/3	1973
Goldman Sachs Industrial Metals Index	8 1/3	1977
Subtotal of Commodity Component	25	

Note: The index has been backdated to 1970. Prior to 1973, there were no commodity contracts on hard assets with sufficient liquidity to qualify for inclusion in the Goldman Sachs indexes. Therefore, the index is all equity for the period 1970-1972. Since the NAREIT didn t exist prior to 1972, for those years the four MSCI series each receive a weighting of 25 percent. In 1972, each of the equity indexes (including the NAREIT) received a 20 percent weighting. The historical weightings of commodity positions for the Index change as each commodity sub-index became available. The precious metals sub-index was first available in 1973 and until 1977 represented all of the commodity component (25 percent of the overall index). The industrial metals sub-index originated in 1977, and from that point until 1983, these two sub-indexes each represent 50 percent of the commodity component. Energy-related commodities began in 1983. Since 1983, each of the sub-indexes represents 33 percent of the commodity component.

2. The commodity futures used in the index are fully-collateralized futures. Collateralized futures contracts consist of an unleveraged long commodity futures position held for a period of time, while using U.S. Treasury bills as 100 percent collateral. Returns on collateralized futures contracts are derived from three sources: change in market value of the contracts, interest from the Treasury bills used as collateral, and any gain or loss made when rolling from the maturing futures contract into the next available month s contract.

3. From 1973 to 1977 precious metals comprise the entire commodity component. This was a period where precious metals experienced high returns relative to other asset classes. This does not compromise the results because (a) the risk associated with the returns was also high, therefore, the returns relative to risk are appropriate and (b) inflation was high during this period and had other contracts been available it is likely that they would have followed suit.

Hard Assets in Efficient Portfolios

Figure 1 shows how the inclusion of hard assets can potentially improve performance. The lower efficient frontier is derived from an optimization that excludes hard assets from consideration. The upper frontier is derived from an optimization that allows hard assets to be included. On each frontier we have identified three portfolios that roughly correspond to portfolios with standard deviation levels of 8 percent (low risk), 12 percent (medium risk), and 18 percent (high risk). The resulting portfolios and their expected returns, standard deviations, and Sharpe ratios are shown in Table 2.

The portfolios exhibited in Table 2 are not directly on the upper efficient frontier for two reasons.

1. Allocations to asset classes in the portfolios were held at 5 percent increments.

2. In selecting the "With Hard Assets" portfolios listed in Table 2, we did not blindly select points off the efficient frontier that included hard assets. Portfolios off the frontier are efficient only if our forecasts for expected return, risk, and correlation are perfectly accurate. Since forecasts inevitably have estimation error, we have modified the efficient portfolios to take into account the possibility of the optimization inputs being incorrect. The end result of this "sensitivity analysis" is to produce asset allocations that are not just optimal in any given scenario, but close to optimal under a variety of scenarios.

In each case, the hard assets portfolio beats the hard assets-free portfolio based on the traditional Sharpe ratio.[3] By including the GHAI in the allocation investors can potentially improve the reward/risk ratio in each of the sample low, medium, and high risk portfolios. This forecast implies that including hard assets in a portfolio should increase expected returns and reduce portfolio risk.

Notice that the allocation to hard assets does not occur at the expense of a single asset class, but from a reduction in allocation to several asset classes. In fact, when evaluating the efficient frontier points themselves (not presented here), at each risk level, allocations to the S&P 500, international stocks and Treasury bills are decreased when hard assets are included in the portfolios. This might imply that the unique risk/reward profile of the GHAI makes it a particularly useful diversification tool for domestic large-cap stocks, international stocks, and cash.

Figure 1: Efficient Frontier with and without Hard Assets

Table 2: Asset Allocations With and Without Hard Assets

	Low Risk		Medium Risk		High Risk	
Asset Class	With Hard Assets	Without Hard Assets	With Hard Assets	Without Hard Assets	With Hard Assets	Without Hard Assets
Hard Assets	10	0	20	0	25	0
U.S. Small Stocks	5	5	10	10	15	15
U.S. Large Stocks	15	15	25	25	35	45
International Stocks	10	10	10	15	25	25
U.S. Interm-Term T-Bonds	35	50	30	45	0	15
U.S. Treasury Bills	25	20	5	5	0	0
Expected Return	8.6	8.1	10.9	10.1	14.1	13.3
Standard Deviation	7.7	7.8	11.5	11.5	17.4	17.7
Sharpe Ratio	0.55	0.47	0.56	0.49	0.55	0.50

Note: Sharpe ratios computed using expected returns for the portfolios and Treasury-bills (see Table 5) and expected portfolio standard deviations.

Index Performance

Figure 2 shows the growth of a $1 investment made on December 31, 1969, in various asset classes including the GHAI.[4] The index under-performed all of the pure equity investments and U.S. intermediate-term Treasury bonds,[5] but outperformed U.S. Treasury bills and inflation.

Summary statistics on these assets are in Table 3. Not surprisingly, the higher-returning asset classes generally have higher risk. When viewed in isolation, the GHAI has been roughly as volatile as stocks, with a standard deviation of 17.5 percent over the 1970 to 1998 period although less volatile over the 1982 to 1998 period.

Table 3 also shows asset class performance in high (1970-81) and low (1982-98) inflationary periods, respectively. Note that the GHAI exhibits a slightly higher return and standard deviation in the inflationary scenario. Other equities and bonds suffer in the inflationary period with, for the most part, lower returns and higher volatility, while cash keeps pace with inflation.

Figure 2: Growth of $1 Investment, 1970-1998

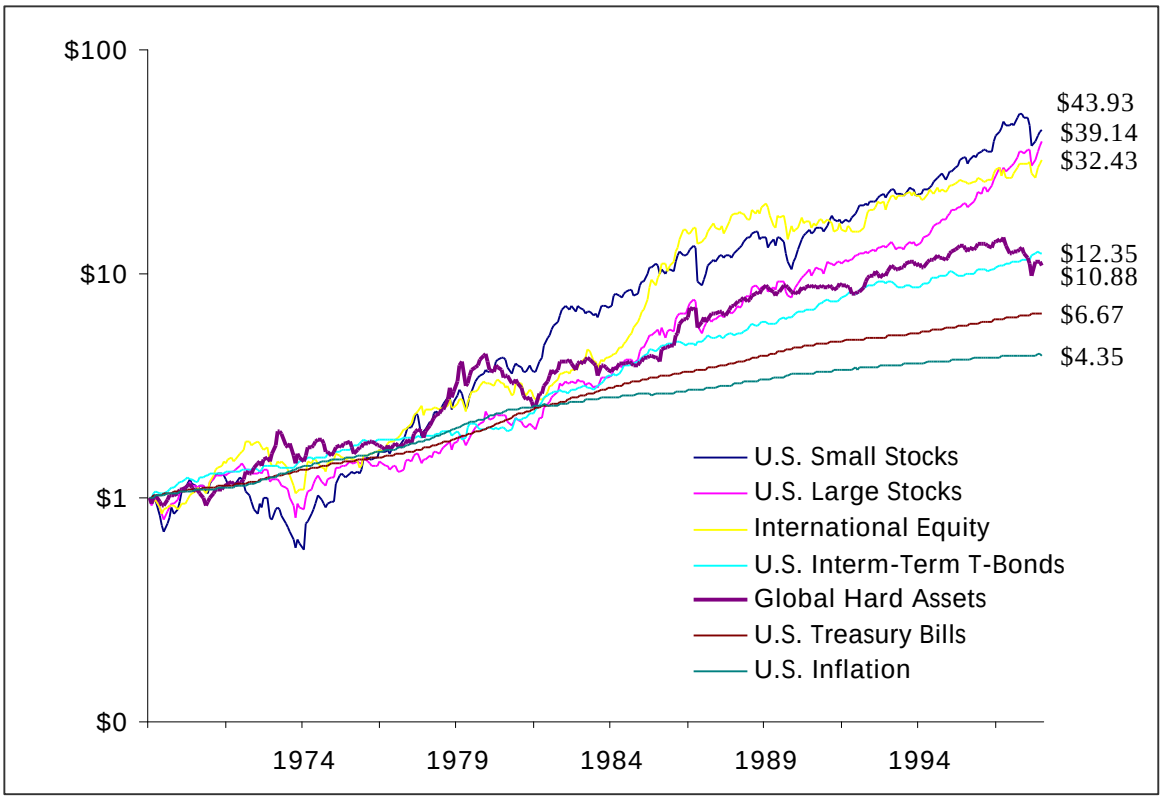


Table 3: Historical Returns and Standard Deviations

Asset Class	Entire Period 1970 - 1998 Compound Annual Return	Entire Period 1970 - 1998 Standard Deviation	High Inflation 1970 - 1981 Compound Annual Return	High Inflation 1970 - 1981 Standard Deviation	Low Inflation 1982 - 1998 Compound Annual Return	Low Inflation 1982 - 1998 Standard Deviation
Global Hard Assets	8.6	17.8	10.3	20.3	7.4	15.9
U.S. Small Stocks	13.9	23.2	12.0	27.1	15.3	20.0
U.S. Large Stocks	13.5	17.6	6.9	17.2	18.4	17.7
International Stocks	12.7	19.3	10.2	17.8	14.6	20.4
U.S. Interm-Term T-Bonds	9.1	6.5	6.9	7.4	10.6	5.6
U.S. Treasury-Bills	6.8	0.8	7.4	1.0	6.3	0.7
U.S. Inflation	5.2	1.2	7.9	1.3	3.3	0.8

Note: Compound annual returns and standard deviations reported are annualized based on monthly returns. Although there are sufficient data to use annual points for the period 1970-1998, we chose to show monthly-annualized figures to increase comparability across time periods.

Hard Assets Correlation Coefficients

Equities

Table 4 shows the correlation coefficients of annual returns between the GHAI and other asset classes over the 1970-1998 period. Correlation coefficients of the separate equity and commodity components are also in Table 4. The GHAI has had low correlation coefficients with stocks, ranging from 0.16 to 0.22.

As expected, the equity component of the index exhibits higher correlation coefficients with the stock markets than does the commodity component. However, in general, the correlation coefficients of the equity component with the other equity indexes are moderate, ranging from 0.33 to 0.39. This is likely a result of the specific nature of the industries that comprise the equity component, and the fact that the companies are not just domiciled in the United States.

The commodity component of the GHAI exhibits negative correlation coefficients with stocks (-0.29 to -0.19). This illustrates the added benefits from including commodities in the index. They appear to offer diversification benefits by hedging the risk associated with stocks.

Table 4: Correlation Coefficients of Annual Total Returns for Hard Assets and Other Asset Classes, 1970-1998

	Global Hard Assets Index	Hard Assets Equity Component	Hard Assets Commodity Component
U.S. Small Stocks	0.16	0.33	-0.22
U.S. Large Stocks	0.14	0.35	-0.29
International Stocks	0.22	0.39	-0.19
U.S. Interm-Term T-Bonds	-0.17	-0.05	-0.31
U.S. Treasury-Bills	0.03	0.00	0.07
U.S. Inflation	0.33	0.19	0.47

Note: Correlation coefficients between the commodity component of the GHAI and other asset classes are measured over the period 1973-1998.

Fixed-Income

The GHAI, as well as both the equity and commodity components, had negative correlation coefficients with intermediate bonds. In fact, the correlations with intermediate U.S. government bonds are the lowest of all the asset classes. However, this correlation is not particularly stable. Looking at the correlation of returns over rolling 60-month windows, the correlation of the Index and U.S. intermediate-term Treasury bonds ranged from 0.25 to -0.25.

Inflation

The correlation between the GHAI and inflation was 0.33 for the 1970 to 1998 period. This indicates that the index has acted, to a limited extent, as an inflation hedge. Not surprisingly, the commodity component is fairly strongly correlated with inflation. The equity component is also positively correlated with inflation, but to a lesser degree. Many studies have documented that broad market equity indexes tend to be negatively correlated with inflation.[6] The fact that the equity portion of the index has a slight positive correlation with inflation is due to the nature of the companies in the index (e.g., mining).

Year-by-Year Performance of the Hard Assets Index

An examination of the performance of the GHAI relative to other assets on a year-to-year basis helps illustrate the diversification potential of the index. Figure 3 is a scatter plot of the annual total returns on the index and the S&P 500. Some of the more interesting years are labeled. There are only four years where both the GHAI and the S&P 500 had negative returns and in only one of those years (1974) was the decline severe. In that year the GHAI declined approximately 7 percent while the S&P 500 dropped more than 26 percent.

The performance of the index in 1973 and 1979 illustrates the impact of economic shocks. Each of these years included oil price shocks, and in each year hard assets outperformed the stock markets, returning 32.3 percent and 70.0 percent in the respective years.

Following the 1973-1974 falloff in stocks was a bull market recovery from 1975-1976. In 1975 and 1976 S&P 500 returns soared past 20 percent while hard assets returned a respectable 10 percent. The out-performance of the S&P 500 might have been due to the fact that hard assets did not fall nearly as much as the S&P 500 during 1974.

The year 1987 is interesting in that we see that the crash of October wiped out most of the large gains accumulated by the S&P 500 during that year. The GHAI was mostly immune to those effects and had a return of over 30 percent.

Figure 4 shows a similar analysis comparing intermediate bonds and GHAI. Some of the more interesting years were 1981, a decent year for bonds, but not for hard assets. 1982 was an excellent year for bonds as interest rates declined. But that didn t translate to superior returns for hard assets, nor would we expect it to. On the other hand, 1994 was a period of rapidly rising rates, which is reflected in the fact that that year was the only negative total return for bonds, but a decent year for hard assets. 1979 was the best year for hard assets as the asset class rode the wave of high actual and expected inflation, an ominous combination for bond investors.

As an aside, the results displayed in Figures 3 and 4 suggest that hard assets are an effective diversifier for stocks and bonds, but may be better for bonds. This is consistent with the correlation results from Table 4.

Figure 3:Scatterplot of Annual Returns for the S&P 500 and the Global Hard Assets Index

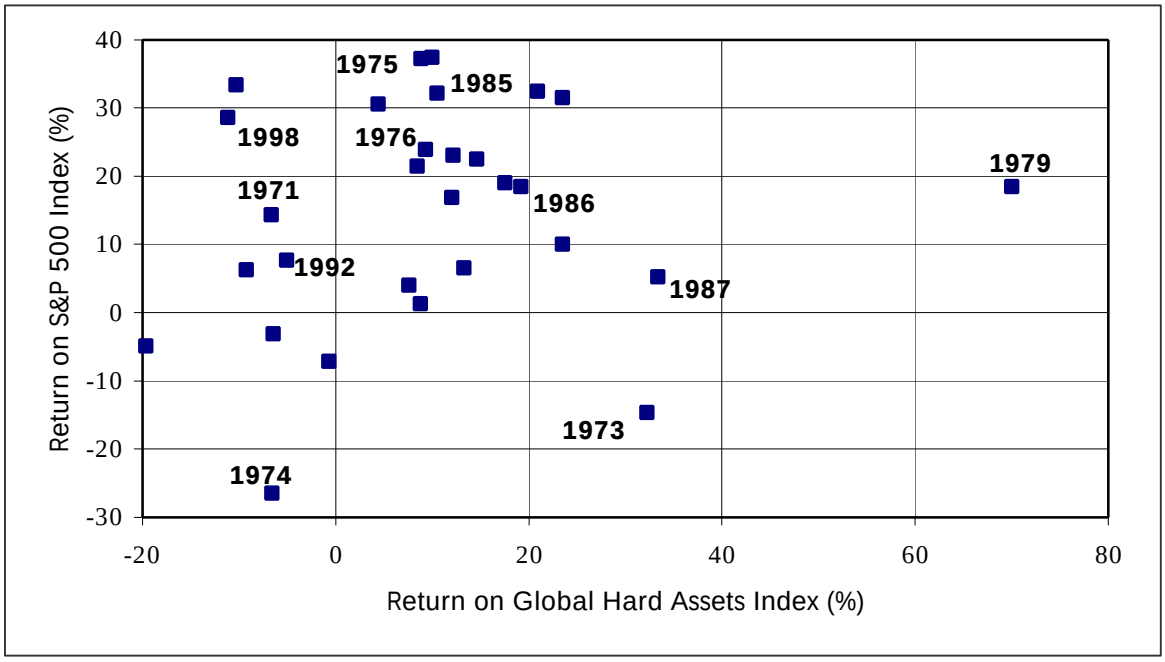
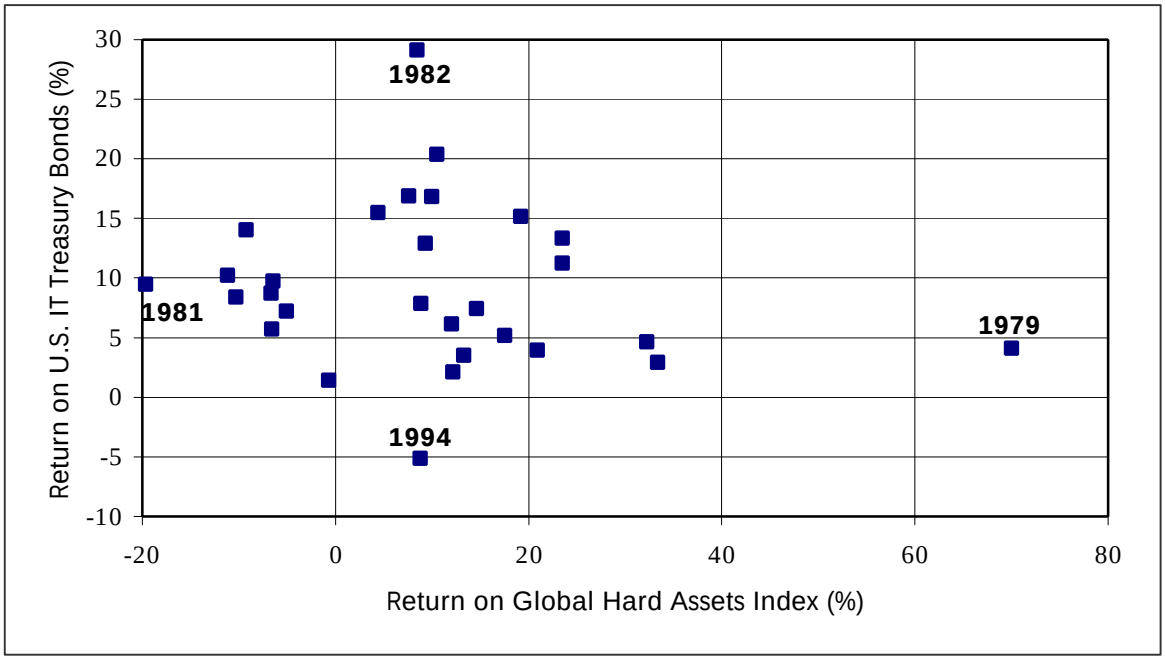


Figure 4: Scatterplot of Annual Total Returns for the U.S. Intermediate-Term Treasury Bonds and Global Hard Assets



Determining the Role of Hard Assets in an Asset Allocation

The role that an asset class plays in an overall strategy is dependent on three components: risk, return, and correlation. In particular, we need to incorporate correlation into our analysis. Incorporating correlation allows us to determine the contribution of an asset class to overall portfolio risk, which is what investors care about. Historically, hard assets have had low correlation coefficients with other asset classes, thus they may do well when other assets do poorly.

We believe, however, the historical data presented is important only insofar as it helps to develop better expectations for future return, risk, and correlation coefficients for the different asset classes. What we are concerned with is not so much what happened, but what can reasonably be expected to happen over one's investment horizon. Simply plugging the historical numbers into an optimization package will tell us the role, if any, that hard assets would have played in an efficient asset allocation. However, unless those historical numbers are exceptional forecasts for the future, their usefulness is reduced.

For our study, we selected the following asset classes used in the historical comparison for use in optimization:

1. U.S. small stocks
2. U.S. large stocks (S&P 500)
3. International stocks (MSCI EAFE)
4. Intermediate-term U.S. Treasury bonds
5. U.S. Treasury bills; and

6. Global hard assets (GHAI)

Table 5: Expected Returns and Standard Deviations

Asset Class	Expected Return	Standard Deviation
Hard Assets	11.82	20.89
U.S. Small Stocks	16.69	30.01
U.S. Large Stocks	13.85	20.26
International Stocks	15.00	27.32
U.S. Interm-Term T-Bonds	5.51	6.77
U.S. Treasury Bills	4.43	2.66

Table 6: Expected Correlation Coefficients

Asset Class	Hard Assets	U.S Small Stocks	U.S. Large Stocks	International Stocks	Interm-Term T-Bonds	30 Day Treasury Bills
Hard Assets	1.00	0.16	0.14	0.22	-0.17	0.03
U.S. Small Stocks	0.16	1.00	0.86	0.38	0.22	0.00
U.S. Large Stocks	0.14	0.86	1.00	0.47	0.36	-0.10
International Stocks	0.22	0.38	0.47	1.00	0.06	-0.20
U.S. Interm-Term T-Bonds	-0.17	0.22	0.36	0.06	1.00	0.23
U.S. Treasury Bills	0.03	0.00	-0.10	-0.20	0.23	1.00

Our expected return, standard deviation, and correlation assumptions are provided in Tables 5 and 6, respectively. The procedures we used to create these inputs are briefly described below.[7]

Expected Return

The general approach when estimating the expected return for an equity asset class is to determine the investment horizon of an investor and then determine what is the risk-free rate for that time period. We assume our investor has a 20-year horizon and estimate the risk-free return as being the yield on a 20-year coupon bond. As of March 26, 1999 this was 5.88 percent.

The expected return is then the risk-free rate plus an equity risk premium. The equity risk premium is assumed to be 8.0 percent for U.S. large-cap stocks,[8] 10.8 percent for U.S. small-cap stocks,[9] and 9.1 percent for international stocks.[10]

For fixed-income expected returns, we use the risk-free rate plus a horizon premium. This premium is our estimate of the compensation that investors expect to receive in exchange for bearing the higher volatility of bonds with longer duration. Since we are using a risk-free rate based on a 20-year bond, the horizon premium for intermediate-term bonds and bills is subtracted from the yield of the long bond.[11]

Neither of the approaches described above are fully applicable for hard assets because the equity and commodity components of the index behave so differently. We therefore model the expected return for each component separately and then make use of the 75/25 weighting to estimate the expected return for the entire index as a weighted average of the components.

The expected return of the equity component is estimated by multiplying the beta of the equity component of the GHAI relative to a world stock index by the global equity risk premium;[12] to this product is added the risk-free rate. The result of these calculations is an expected return of 13.50 percent for the equity component.

The return on commodities is logically and empirically related to inflation. Our model for estimating the expected total return on commodities is to take the expected real return to commodities and add to that the expected inflation rate.

We used the historical difference in the arithmetic mean total returns on the commodity component of the GHAI and U.S. inflation from 1983 to 1998 as our estimate for the expected real return on commodities. Using monthly-annualized data, this real return estimate is 4.20 percent. We selected this period because only over this period was the full commodity index available (i.e., prior to that the index consisted only of gold and copper). The inclusion of the oil and gas components substantially changed the nature of the index and we believe using a real return over this more recent period is a more reasonable reflection of how the commodity portion will behave in the future.

The Ibbotson twenty-year inflation forecast is 2.6 percent.[13] Summing the historical real return and expected inflation gives an expected return of 6.8 percent for the commodity component.

Applying the component weights of 75 percent and 25 percent to the respective expected returns leads to an expected return on the GHAI of 11.82 percent.

Standard Deviation and Correlation Coefficients

The expected standard deviations and correlation coefficients used in optimization inputs are based on historical standard deviations and correlation coefficients over a period of time that we believe reflects the range of possible outcomes for the future.

For U.S. large- and small-cap stocks we have total return data dating back to 1926 and we use the standard deviation of annual returns over this entire time period to compute our estimated standard deviation of the future. The correlation between these two asset classes is also estimated using this time period.

Return data on EAFE dates back only to 1970. We believe using a standard deviation from this time period is probably an underestimate. To adjust this result we divide the EAFE standard deviation from 1970 to 1998 by the S&P 500 standard deviation of the same period and then multiply that ratio by the standard deviation of the S&P 500 from 1926 to 1998. The expected correlation of international stocks is assumed to be the

correlation of EAFE with our proxies for the other asset classes over the 1970 to 1998 period.

The expected standard deviations for fixed-income classes are estimated in the same manner as used with U.S. equities, but we use the time period from 1970 to 1998. We only use the post-1970 period since the volatility of interest rates increased sharply at about that time and shows no sign of abating. The correlation of the bonds and bills is estimated to be the actual correlation of these asset classes with those other asset classes over the 1970 to 1998 period.

For the GHAI we chose to prepare separate estimates for the equity and commodity components. The estimate for the index would then be prepared using the standard portfolio standard deviation formula.[14] The standard deviation for the equity component is calculated using the same ratio method as was used for international stocks. The result is an expected standard deviation of 21.75 percent.

The standard deviation of the commodity component is not adjusted as a suitable benchmark adjustment is not available and the historical standard deviation of 30.85 percent for the 1973 to 1998 period appears reasonable. The standard deviation for the GHAI as derived is then 20.89 percent.

The expected correlation between the GHAI and other asset classes is assumed to be the same correlation that was observed using the 1970 to 1998 data.

Conclusion

Hard assets offer investors an attractive option for portfolio diversification. Including hard assets in a portfolio can potentially increase returns and reduce risk, given the asset classes evaluated in this study. Furthermore, allocations to hard assets should help risk-averse investors further diversify their portfolios without impacting their expected return. Using Sharpe ratio analysis, portfolios including hard assets have been shown to offer better performance than those without hard assets. Hard assets alone have not presented an opportunity for extraordinary returns, nor do they eliminate portfolio risk. However, as part of a diversified portfolio, their low correlation coefficients with other asset classes and positive correlation with inflation offer some protection against adverse market movements.

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[1] See Lummer and Siegel [1993] and Ankrim and Hensel [1993].

[2] In this study we devise three portfolios of varying risk levels. These portfolios may not be appropriate for all investors.

[3] The Sharpe ratio is a return-to-risk measure. It is a ratio of the excess return over the risk-free rate to the standard deviation. It was first used by Sharpe [1966].

[4] Figure 2 assumes that all cash flows generated from each asset class are reinvested and that no taxes or transactions costs are paid.

[5] Our proxy for U.S. small-cap stocks is the CRSP (Center for Research in Security Prices) NYSE 6-8 index. This index is composed of stocks listed on the New York Stock Exchange that rank within 6th, 7th, and 8th deciles when those stocks are ranked by their market capitalization. This index has the advantage of having data back to 1926 (unlike the Russell 2000, which is inadequate for our review, as it only dates to 1979) and has a very high correlation with that index (0.95 using annual data over the 1970-1998 period).

[6] The first empirical works in this area appear to be Jaffe and Mandelker [1976], Bodie [1976], and Nelson [1976].

[7] A more complete description can be found in Lummer, Riepe, and Siegel [1994].

[8] The equity risk premium is estimated by subtracting the arithmetic mean of the annual income rate of return of Ibbotson Associates 'U.S. Long-term Treasury Bond Index from the total return on the S&P 500 for the period 1926 to 1998.

[9] This higher premium reflects the empirical result that small stocks tend to have higher returns than large-cap stocks. The equity risk premium on the S&P 500 is increased by the difference in the arithmetic return of the small-cap index over the 1926 to 1998 period and the arithmetic return of the S&P 500. The expected return for small-cap equities can be thought of as being the sum of the risk-free rate, large-cap equity risk premium, and small-cap premium.

[10] The equity risk premium for international stocks is calculated in a slightly different manner. Using regression analysis, we compute the beta of international equities against the world and the beta of the S&P 500 against the world. We multiply the U.S. equity risk premium by the ratio of the international vs. world beta over the S&P 500 vs. world beta to arrive at the international equity risk premium.

[11] The horizon premium is estimated as the difference between the arithmetic means of the income returns of the long bond and intermediate-term U.S. government bonds and 30-day U.S. Treasury bills over the period 1970-1998. The horizon premium for intermediate-term bonds is -0.37 percent and -1.45 percent for Treasury bills. Bond income return is defined as the total return realized if the yield on the bond return had remained constant over the time period. The bond income return is used because it is the return attributable solely to the bond's yield, which is an unbiased measure of market expectations.

[12] The beta of 0.86 was estimated by regressing the monthly total returns from January 1970 to December 1998 of the equity component of the Global Hard Assets Index on the MSCI World Total Return Index. The global equity risk premium is calculated by dividing the U.S. equity risk premium by the beta of the S&P 500 on the MSCI World Index. For the period 1970-1998, this beta was 0.90; combined with a U.S. equity risk premium of 7.97 percent, the global equity risk premium is estimated to be 8.86 percent.

[13] Ibbotson Associates forecasts long-term inflation by subtracting a forecast of the long-term real risk-free rate and an estimate of the maturity premium from the observed current 20-year Treasury-bond yield. The most recent estimate is from Ibbotson Associates, *Cost of Capital Quarterly 1999 Yearbook*, Chicago, 1999.

[14] This equation requires the standard deviations of each component, the weights of each component and the correlation between components. The correlation is assumed to 0.44 which is the historical correlation (calculated using annual total returns from 1973-1998) between the two components.